LESSON 4

Comparing Proportional Relationships

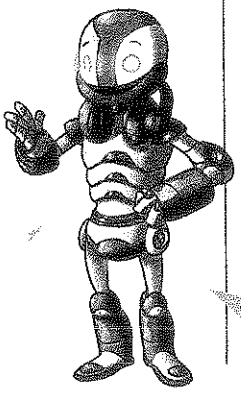
PLUG IN Unit Rate and Slope of a Line

A **unit rate** is a ratio that compares a quantity to 1.

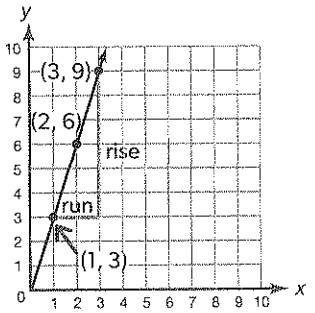
x	y	$\frac{y}{x}$
1	3	$\frac{3}{1}$
2	6	$\frac{6}{2} = \frac{3}{1}$
3	9	$\frac{9}{3} = \frac{3}{1}$
4	12	$\frac{12}{4} = \frac{3}{1}$
5	15	$\frac{15}{5} = \frac{3}{1}$

In this table, the unit rate is $\frac{3}{1}$.

I see! When written as a fraction, the unit rate has a denominator of 1.



The **slope** of a line is a ratio of the change in its y-coordinates (the *rise*) to the change in its corresponding x-coordinates (the *run*).



By the points (1, 3) and (3, 9), the slope of the line is $\frac{9-3}{3-1} = \frac{6}{2} = \frac{3}{1}$.

I get it! The unit rate and the slope are equivalent!

unit rate
a ratio that compares a quantity to 1

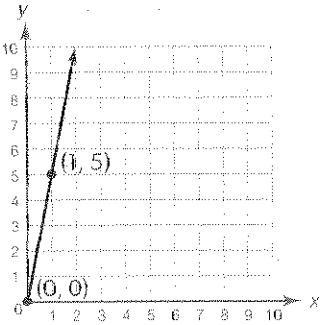
slope
a ratio of the change in y-coordinates (*rise*) of a graph to the change in corresponding x-coordinates (*run*)

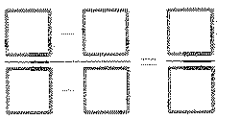
DISCUSS Review the table and graph. Which points on the line can you use to calculate its slope?

A You can find the slope to determine the unit rate of a proportional relationship.

DO The graph represents a proportional relationship. What is the unit rate of the graph?

1. Use the labeled points to find the slope of the line.
2. Write the slope as a ratio in simplest form.
3. The unit rate is equivalent to the slope.





The unit rate is $\frac{\square}{\square}$.

B You can find the unit rate for a real-world situation.

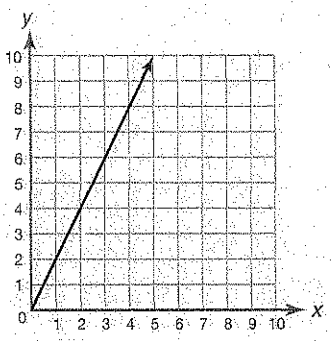
DO

This graph represents the total cost of cheese based on the number of pounds purchased. This relationship is proportional.

Make sure you subtract the x-values in the same order that you subtract the y-values from the two points.

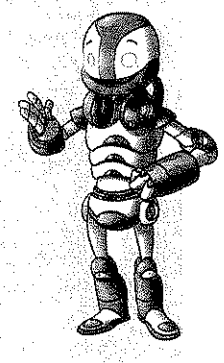
Find the cost per pound, or unit rate.

1. Pick two points on the line and find the slope.
2. Write the slope as a ratio in simplest form.
3. The unit rate is equivalent to the slope of a proportional relationship.



$\frac{\square - \square}{\square - \square} = \frac{\square}{\square}$

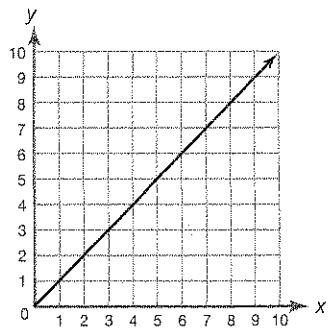
The unit rate is _____ per pound.



PRACTICE

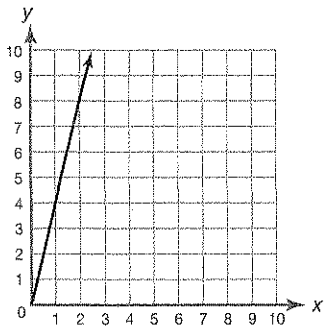
Find the unit rate of each proportional relationship.

1



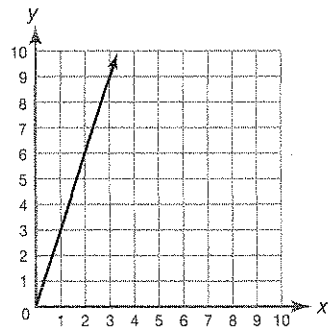
The unit rate is _____.

2



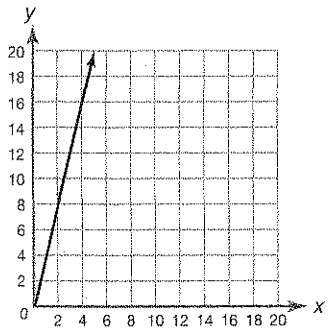
The unit rate is _____.

3



The unit rate is _____.

4



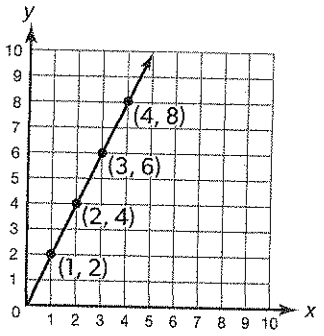
The unit rate is _____.

POWER UP Graphing a Proportional Relationship

In a proportional relationship, the ratios of the quantities compared are the same.

Weight (in pounds)	Cost (in dollars)	Ratio
1	2	$\frac{2}{1}$
2	4	$\frac{4}{2} = \frac{2}{1}$
3	6	$\frac{6}{3} = \frac{2}{1}$
4	8	$\frac{8}{4} = \frac{2}{1}$

You can use the values in the table as coordinate pairs to create a graph of the proportional relationship.



I see! Since each of the ratios can be written as $\frac{2}{1}$, the table shows a proportional relationship.

I remember! The graph of a proportional relationship is a line that passes through the origin.

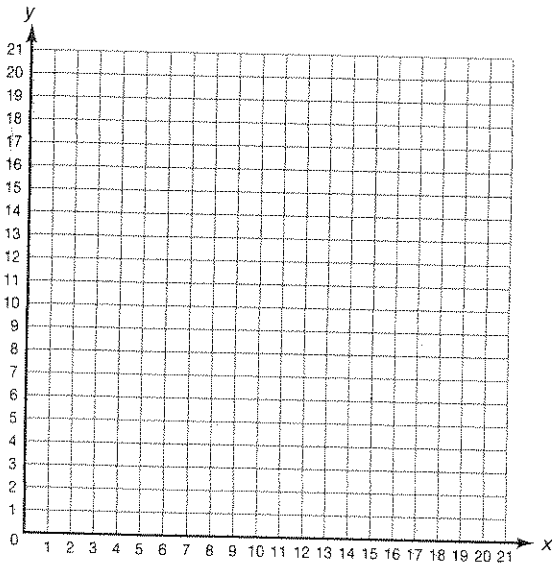
DISCUSS Ming bought 10 pounds of meat for $30. If this relationship between cost and weight were graphed, how would she know if the relationship was proportional?

A You can identify a proportional relationship by graphing a table of data.

DO Graph the relationship shown in the table.

1. Plot each point.
2. Connect the points with a line.
3. Determine if the line passes through the origin.

x	y
1	4
2	8
3	12
4	16
5	20



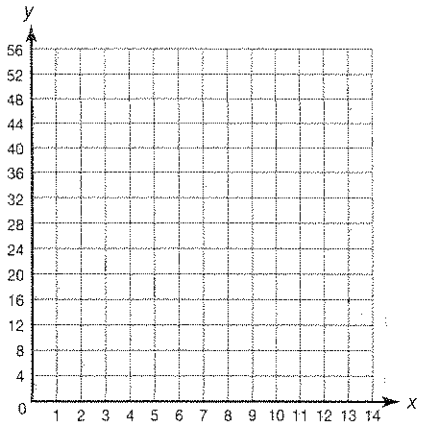
The line includes the origin. The table represents a proportional relationship.

B You can graph a proportional relationship using a verbal description.

DO Tamara earns $8 per hour for babysitting. Complete the table and graph this relationship.

1. Complete the table.
2. Graph the data by plotting each point.
3. Draw a line that includes each point.

Hours	Amount Earned (in dollars)
1	
2	
3	
4	
5	



DISCUSS Oliver knows the graph above represents a proportional relationship. He wonders if a line parallel to this graph through (1, 9) is a proportional relationship as well. How would you explain the answer to him?

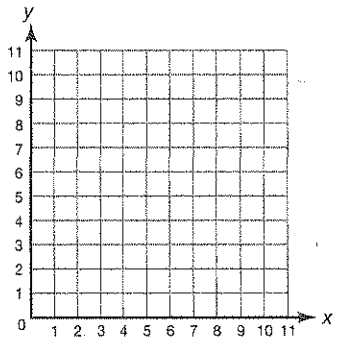
You can sketch the line Oliver is thinking of to help you.

PRACTICE

Graph each proportional relationship.

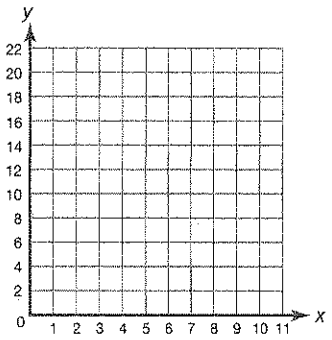
1

x	y
2	1
4	2
6	3
8	4
10	5



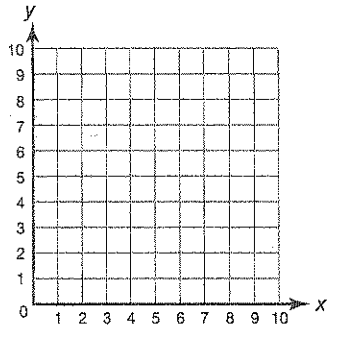
2

x	y
0	0
1	5
2	10
3	15
4	20



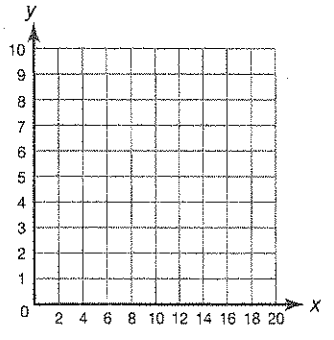
3

x	y
1	1
2	2
3	3
4	4
5	5



4

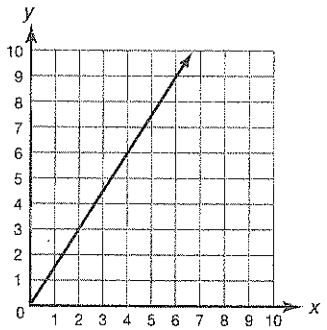
x	y
3	1
6	2
9	3
12	4
15	5



READY TO GO Comparing Proportional Relationships

Proportional relationships can be represented using a table or graph.

x	2	4	6	8	10
y	3	6	9	12	15



The unit rate is $\frac{1.5}{1}$.

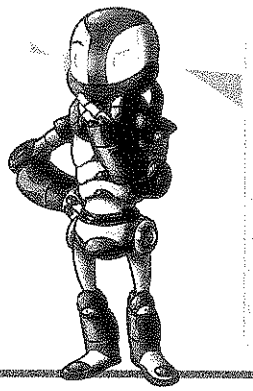
The slope is $\frac{3}{2}$ or 1.5.

Another way to represent a proportional relationship is with an equation.

All proportional relationships have the form $y = kx$, where k is any nonzero number and represents the unit rate (or slope).

The equation is $y = 1.5x$.

I know this is a proportional relationship because the ratio $\frac{y}{x}$ is constant in the table, and because the graph is a line that passes through the origin.



I see! To find the unit rate of a proportional relationship expressed as an equation, I can just look at the coefficient of x.

DISCUSS Compare the proportional relationship $y = 3x$ to $y = 1.5x$. Which has a greater unit rate? How do you know?

LESSON LINK

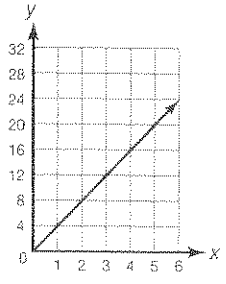
PLUG IN

You can determine the unit rate of a proportional relationship by finding the constant ratio $\frac{y}{x}$ in a table of values or the slope of its graph.

x	1	2	3	4	5
y	4	8	12	16	20

POWER UP

You can graph a proportional relationship, which is represented by a line that passes through the origin.



GO!

I see! I can compare proportional relationships represented as graphs, tables, or equations by finding the unit rate.

I see! The words "constant rate" let me know that these relationships are proportional.

WORK TOGETHER

You can compare two proportional relationships by finding their unit rates.

- Find Sherine's unit rate.
- Find Devon's unit rate.
- Compare the unit rates.

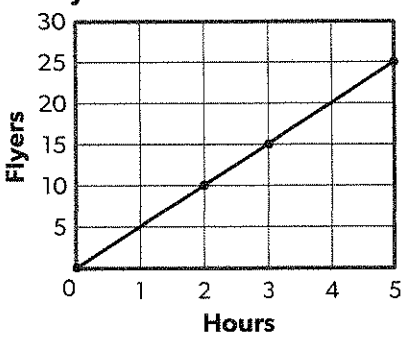
Sherine and Devon each handed out flyers at a constant rate. Sherine made a table of the number of flyers she handed out, and Devon made a graph. Who handed out flyers at a greater rate?

Sherine's Table

Flyers Passed Out over Time

Hours	Flyers
0	0
1	7
2	14
3	21
4	28

Devon's Graph



The unit rate is the change in quantity per unit of measure.

$$\text{Sherine's rate} = \frac{\text{change in flyers}}{\text{change in hours}} = \frac{14 - 7 \text{ flyers}}{2 - 1 \text{ hours}} = \frac{7 \text{ flyers}}{1 \text{ hour}} = 7 \text{ flyers per hour}$$

The slope of a graph is equal to the unit rate.

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope of Devon's graph} = \frac{15 - 10 \text{ flyers}}{3 - 2 \text{ hours}} = \frac{5 \text{ flyers}}{1 \text{ hour}} = 5 \text{ flyers per hour}$$

Sherine's rate is greater.

A You can compare proportional relationships that are represented in different ways.

DO

Dwayne used the equation $y = 3.25x$ to represent the total cost when buying x gallons of gasoline in City A. Dena created a table to represent the total cost when buying x gallons of gasoline in City B. Which city sells gasoline at a lower unit rate?

x	2	4	6	8	10
y	6.40	12.80	19.20	25.60	32.00
$\frac{y}{x}$					

1. Identify the unit rate for City A. The equation $y = 3.25x$ represents City A.
2. Calculate the unit rate for City B. The unit rate is _____.
3. Compare unit rates. _____ > _____. City _____ sells gasoline at a lower unit rate.

DISCUSS

If you know the table of values for a proportional relationship, how could you express the relationship as an equation?

PRACTICE

Choose the proportional relationship with the greater unit rate.

REMEMBER
The slope of a graph equals the unit rate.

1 The graph shows the number of shopping bags shipped to a bookstore each month.

The bookstore also gets a shipment of 750 books every month.

Which item is delivered at a greater unit rate, shopping bags or books?

_____

2 The table shows the number of sea lions seen by tourists over time.

Sea Lion Sightings over Time

Hours	Number Seen
0	0
2	32
4	64
6	96

If the biologist saw 13 sea otters per hour, the animal that she saw at a greater rate is the

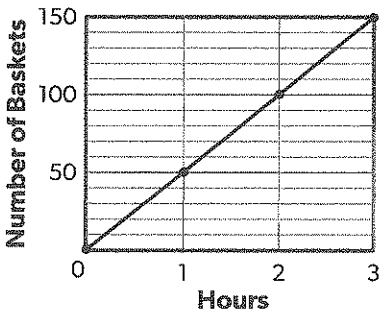
_____.

HINT
You can compare two proportional relationships directly if they're in the same form.

3 The graph shows the number of plums picked over time.

Fruit Harvest at Williams Orchards

Plums Picked over Time



Peaches Picked over Time

Hours	Number of Baskets
0	0
2	126
4	252
6	378

The equation $y = 63x$ represents the number of peaches picked, y, in x hours.

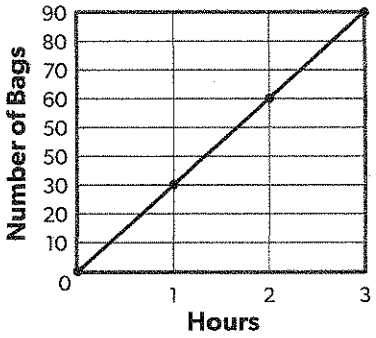
The fruit picked at a greater rate is _____.

Choose the proportional relationship with the greater rate.

4 The table and graph show the number of food items sold over time at basketball games.

Food Sold at Basketball Games

Peanuts Sold over Time



Hot Dogs Sold over Time

Hours	Number Sold
$1\frac{1}{2}$	66
2	88
$2\frac{1}{2}$	110

The food sold at a greater rate is _____.

Determine the greater rate in each situation.

I can compare rates more easily if I show the information in these problems in a table.

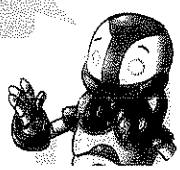
5 Hector raked 1 bag of leaves in 0.5 hours and 3 bags in 1.5 hours. Blaine raked 3 bags of leaves in 1 hour and 6 bags in 2 hours. Who raked leaves at a greater rate? _____

6 A waiter poured 2 gallons of coffee in 2 hours and 6 gallons of coffee in 6 hours. He also poured 1.5 gallons of tea in 3 hours and 2.5 gallons of tea in 5 hours. Which drink did the waiter pour at a greater rate? _____

DISCUSS

Comparing Relationships

I remember! I have to find a constant ratio $\frac{y}{x}$ to show a relationship is proportional.



Luis folded 8 shirts in 10 minutes and 18 shirts in 20 minutes. Carly folded 3 shirts in 5 minutes and 9 shirts in 15 minutes.

Determine if the number of shirts Luis folds per minute represents a proportional relationship. Explain your answer.

What is Carly's unit rate written as a decimal? _____ shirts per minute

Can you compare Luis and Carly's unit rates for folding clothes?

PROBLEM SOLVING

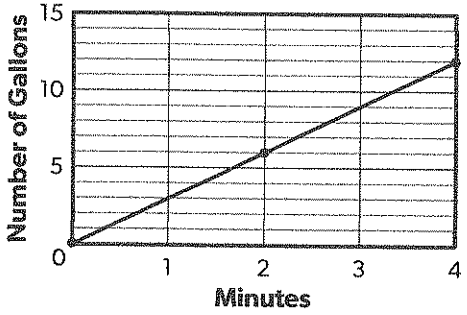
FASTER FAUCET

READ Which faucet should Oliver use to fill the dog bathtub as quickly as possible?

Kitchen Faucet Flow over Time

Minutes	Gallons
0	0
2	4
3	6

Bathroom Faucet Flow over Time



PLAN

- You need to find the faucet with the _____ flow rate.
- You need to know the flow rate of both faucets in _____ per minute.
- Use the table to find unit rate, and use the graph to find slope.

SOLVE Find the flow rate of the kitchen faucet.

$$\frac{\text{change in gallons}}{\text{change in minutes}} = \frac{6 - 4 \text{ gallons}}{\square - \square \text{ minutes}} = \frac{\square \text{ gallons}}{\square \text{ minute}}$$

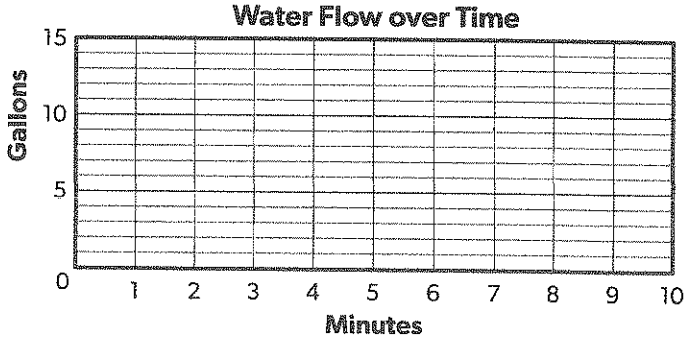
Find the flow rate of the bathroom faucet.

$$\frac{\text{change in gallons}}{\text{change in minutes}} = \frac{\square - \square \text{ gallons}}{\square - \square \text{ minutes}} = \frac{\square \text{ gallons}}{\square \text{ minute}}$$

The faucet with the faster flow rate will fill up the tub faster.

CHECK Graph the flow for both faucets. Include a key. Compare the slopes.

Water Flow over Time

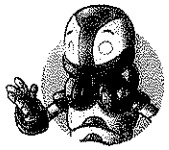


Which faucet should Oliver use? _____

PRACTICE

Use the problem-solving steps to help you.

To find how long it takes the faster runner to run 6 laps, I multiply the unit rate by the number of laps!



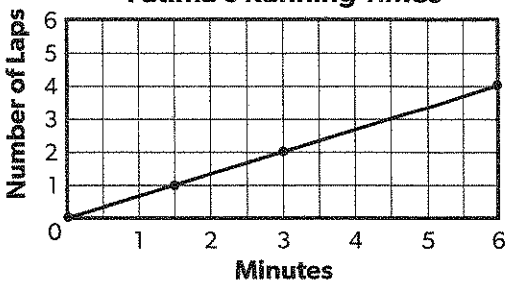
CHECKLIST
- ☐ READ
- ☐ PLAN
- ☐ SOLVE
- ☐ CHECK

1 Keisha and Fatima are running laps around the track.

Keisha's Running Times

Laps	Minutes
0.5	0.8
1.5	2.4
2	3.2

Fatima's Running Times



Who runs at a faster rate? How long would it take the faster runner to run 6 laps?

I can use the unit rate to find the number of sandwiches in a given time.

CHECKLIST
- ☐ READ
- ☐ PLAN
- ☐ SOLVE
- ☐ CHECK

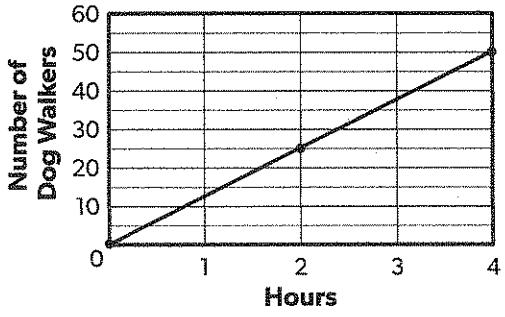
2 Ty and Jamal are making sandwiches together. Ty can make 3 sandwiches in 2 minutes and 6 in 4 minutes. Jamal can make 1 sandwiches in 2 minutes and 2 in 4 minutes.

How many will they have made together in 10 minutes? How many will each have made?

CHECKLIST
- ☐ READ
- ☐ PLAN
- ☐ SOLVE
- ☐ CHECK

3 The number of dog walkers and joggers in a park are shown in the table and graph below.

Dog Walkers Counted over Time



Joggers Counted over Time

Hours	Number of Joggers
0	0
1	12
3	36
5	60

The number of dog walkers and joggers continues to increase at the same rate throughout the day. Will there be more joggers or dog walkers at hour 8? How many will there be of each at hour 8?